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Multi-server Finite Waiting-space Encouraged Arrival Queuing System with Reverse Reneging

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Abstract: It can be seen in every business now-a-days that due to high competition, each firm strives to function smoothly and efficiently to meet customers demand and serve them in the best possible way. Queuing theory plays a very important role here. Queuing models can help firms to understand their performance in advance which can help them prepare well for providing smooth and efficient service to their customers and can enable them to sustain in the long run. As any mistake can prove to be fatal for the reputation of the firm and can adversely affect sustainability of the firm. Firms offer deals and discounts to encourage customers to join the system. Due to encouragement of discounts customers even wait for a longer period of time in the system to avail service. This phenomenon is known as reverse reneging. In this paper we will study a queuing model for any firm encountering encouraged arrivals, reverse reneging and having finite waiting space, operating with multiple parallel servers.

Keywords: Queuing model, reverse reneging, encouraged arrivals, multi-server, finite space.

I. INTRODUCTION

In the current scenario of volatile and competitive business environment, managing business efficiently in order to meet the expectations of the customers and attracting the customers is a very tedious task. As customers have enormous options available now-a-days even for a very basic product. Thus for attracting customers, firms often give huge discounts and other lucrative offers. Due to these offers customers get attracted towards the firm (known as encouraged arrivals coined by (Som and Seth, 2017)), but if firms will not equip themselves to smoothly operate during the offer period to provide customers with ecstatic service experience, reputation of the firm may wane. So firms need to understand their performance level in advance to maneuver service quality and minimizing the waiting time of the customers. Although due to discounts and offers, customers get encouraged to wait for a longer period of time and will have higher patience level (termed as reverse reneging by (Kumar and Som, 2014 & 2015) having less chances of a customer leaving the system), still beyond certain limit few customers might get impatient and leave with negative experience about the firm and can influence others as well with glumness about the firm. Detailed literature of encouraged arrivals, reverse reneging and multi-server queuing models can be seen in (Som and Seth, 2018 & 2019)This paper extends the work by authors in (Som and Seth, 2019) by incorporating multi-server service channels. In this paper we first develop a steady-state queuing model with encouraged arrivals and reverse reneging encountered by any firm having finite waiting space and providing service through various parallel channels. The model is then tested for validity. After establishing the validity of the model we solve the model to derive various probabilistic and performance measures of the firm. The result of these measures can provide a very helpful and effective tool for the firms to access their performance in advance and strategize the operations of the firm for providing smooth and efficient service to the customers and avoid any chaotic situation in future during sale period.

II. MODEL FORMULATION AND SOLUTION

In this section, we formulate the queuing model based on various assumption given below:

- Encouraged arrivals follows Poisson fashion with parameter $\lambda(1+\eta)$, where η represents encouragement factor.
- Service is provided on first come first serve basis through *c* parallel servers.
- System has finite waiting space, say N and cannot accommodate more than N customers at a time.

- Reverse reneging occurs with parameter $\{N-(n-1)\}\xi$, where n represents number of customers present in the system at any time and ξ is reneging parameter which follows exponential distribution.

Using Kolmogorov process, equations of the model in steady-state are given by:

$$0=-\lambda(1+\eta)P_0+(\mu+N\xi)P_1;\quad n=0 \qquad (1)$$

$$0=\lambda(1+\eta)P_{n-1}-\{\lambda(1+\eta)+n\mu+[N-(n-1)\xi\}P_n + \{(n+1)\mu+(N-n)\xi\}P_{n+1};\ 1\le n<c \quad (2)$$

$$0=\lambda(1+\eta)P_{n-1}-\{\lambda(1+\eta)+c\mu+[N-(n-1)\xi\}P_n + \{c\mu+(N-n)\xi\}P_{n+1};\ c\le n\le N-1 \quad (3)$$

$$0=\lambda(1+\eta)P_{N-1}+(c\mu+\xi)P_N;\qquad n=N \qquad (4)$$

Solving Equations (1) – (4) recursively, we get following probabilistic measures:

P_n =Probability of n customers in the system

$$=\begin{cases}\displaystyle\prod_{r=1}^{n}\frac{\lambda(1+\eta)}{[r\mu+\{N-(r-1)\}\xi]}P_0,\ 1\le n\le c; & (5)\\ \displaystyle\prod_{r=1}^{c}\frac{\lambda(1+\eta)}{[r\mu+\{N-(r-1)\}\xi]}\prod_{s=c+1}^{n}\frac{\lambda(1+\eta)}{[c\mu+\{N-(s-1)\}\xi]}P_0,\ c+1\le n\le N\end{cases}$$

By Normality condition, $\sum_{n=0}^{N}P_n=1$, we get

P_0= Probability of no customer in the system

$$=\left[\begin{array}{l}1+\displaystyle\sum_{n=1}^{c}\left\{\prod_{r=1}^{n}\frac{\lambda(1+\eta)}{[r\mu+\{N-(r-1)\}\xi]}\right\}+\\ \displaystyle\sum_{n=c+1}^{N}\left\{\prod_{r=1}^{c}\frac{\lambda(1+\eta)}{[r\mu+\{N-(r-1)\}\xi]}\prod_{s=c+1}^{n}\frac{\lambda(1+\eta)}{[c\mu+\{N-(s-1)\}\xi]}\right\}\end{array}\right]^{-1} \qquad (6)$$

III. MEASURES OF PERFORMANCE

a) Expected system Length: $L_s=\sum_{n=0}^{N}nP_n$

$$=\sum_{n=1}^{c}n\left\{\prod_{r=1}^{n}\frac{\lambda(1+\eta)}{[r\mu+\{N-(r-1)\}\xi]}\right\}P_0 + \sum_{n=c+1}^{N}n\left\{\prod_{r=1}^{c}\frac{\lambda(1+\eta)}{[r\mu+\{N-(r-1)\}\xi]}\prod_{s=c+1}^{n}\frac{\lambda(1+\eta)}{[c\mu+\{N-(s-1)\}\xi]}\right\}P_0 \quad (7)$$

b) Expected system waiting time: $W_s=\frac{L_s}{\lambda(1+\eta)}$

$$=\frac{1}{\lambda(1+\eta)}\left[\begin{array}{c}\displaystyle\sum_{n=1}^{c}n\left\{\prod_{r=1}^{n}\frac{\lambda(1+\eta)}{[r\mu+\{N-(r-1)\}\xi]}\right\}P_0+\\ \displaystyle\sum_{n=c+1}^{N}n\left\{\prod_{r=1}^{c}\frac{\lambda(1+\eta)}{[r\mu+\{N-(r-1)\}\xi]}\prod_{s=c+1}^{n}\frac{\lambda(1+\eta)}{[c\mu+\{N-(s-1)\}\xi]}\right\}P_0\end{array}\right] \quad (8)$$

c) Expected queue waiting time:

$W_q=W_s-\frac{1}{c\mu}$(9)

d) Expected queue length:

$L_q=\lambda(1+\eta)W_q.$ (10)

e) Expected reverse reneging rate:

$R_r=\sum_{n=1}^{N}\{[N-(n-1)\xi\}P_n$(11)

IV. CONCLUSION

Any firm facing challenges like managing heavy rush due to encouraged arrivals where customers are willing to stay even for longer period of time due to lucrative deals being offered by firm, results obtained in this paper can prove to be highly helpful in managing the operations effectively and efficiently. As per the scenario, firm can identify values of the parameters involved and various probabilistic and performance measures obtained in this paper can be easily translated in MS Excel or MATLAB with those identified values to help firm understand its performance well in advance. It can help firm device efficient strategies for smooth functioning of the system, thereby avoiding any chaotic situation they might otherwise face in future due to heavy rush after offering deals and discounts.

Further many more aspects can be studied by extending this paper. Transient solution of this paper can also be studied and cost model can also be incorporated to study financial aspect.

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